Factor Models & Credit Options

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Factor Models: Motivation

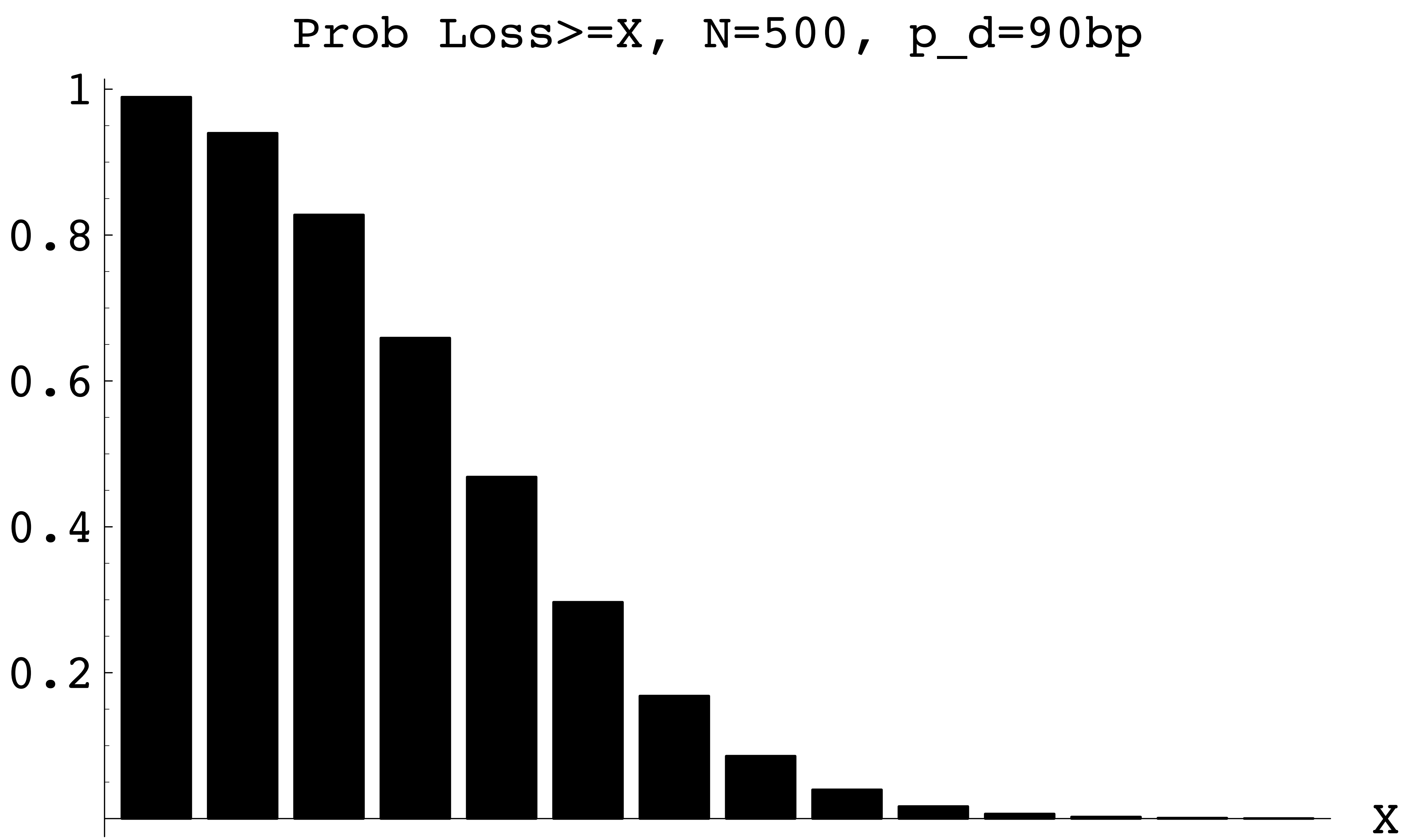
- *Want to measure properties of a portfolio of credit-risky securities*
  - *Expected and unexpected P&L*
  - *Standard deviations or other moments*
  - *Tail sizes*
- *Portfolio members are correlated*

Zero Factor: Moody's Binomial

Portfolio of N uncorrelated companies, each with the same LGD (loss given default) L, identical probabilities p_d of default.

$$P(\text{Total Loss} \geq T) = \sum_{i=\lfloor T/L \rfloor}^{N} \binom{N}{i} p_d^i (1 - p_d)^{N-i}$$

Binomial Results



Use Of Binomial

- *Correlation obviously exists, and LGDs differ; choose N_B and L_B to best capture "equivalent" binomial to actual portfolio*
- *Obviously, there exist choices that will match* (μ,σ)
- *One hopes other important portfolio behavior (e.g. tails) is similar*

Problems With Binomial

- *Choosing N_B and L_B is an art, not a science (done sometimes in collateralized world)*
- *Most important properties such as conditionals and tail values differ between a "real" model and binomial*

Single Factor Models

- *Capture portfolio behavior as being driven by a global factor, plus idiosyncratic elements*
- *Each firm has a driver variable X_n, plus a barrier B_n for X_n whose crossing will result in default (think of assets and liabilities)*

Single Factor: Example Model

A global driver variable Z, plus a unique idiosyncratic variable W_n for each firm and a correlation R_n determines the value of X

$$X_n = R_n Z + \sqrt{1 - R_n^2}\, W_n$$

Default occurs for those n such that

$$B_n > X_n$$

Single Factor Measurements

Simulations from distributions Ω_W, Ω_Z for W and Z are a way of computing e.g. quantiles or tail probabilities. However, the real advantage of single factor models is when the B_n, R_n and LGDs L_n are all resp. equal. Then

$$P[L_{\text{tot}} \leq L_0] = \sum_{k=0}^{\lfloor \frac{L_0}{L} \rfloor} \binom{N}{k} \int_{\mathbb{R}} \left(\Omega_Z \left(\frac{B - RZ}{\sqrt{1-R^2}} \right) \right)^k \left(1 - \Omega_Z \left(\frac{B - RZ}{\sqrt{1-R^2}} \right) \right)^{N-k} d\Omega_Z$$

$$N \longrightarrow \infty, \quad 1 - \Omega_Z \left(\frac{B}{R} - \frac{\sqrt{1-R^2}}{R^2} \Omega_W^{-1} \left(\frac{L_0}{L} \right) \right)$$

Basel II

- *The Basel II risk computations envision a single factor model as above, with the general idea that* Ω_W, Ω_Z *will be normal*
- *The integrations are easy to perform when we make overly ambitious assumptions, but the model has obvious flaws.*
- *Better correlation can be captured at little cost if we are relaxing assumptions anyway*

Multifactor Motivation

- *We want to capture a risk correlation structure of defaults (and/or default risk)*
- *Companies usually just default once. We have no time series to estimate from.*
- *Even default risk has relatively sparse time series data (CDS spreads, ratings).*
- *Directly estimating correlations is hard. Multifactor models have fewer variables.*

Definition of CDS, Asset Swaps

Consider a corporate bond B (issued by a company we call ***O****) with periodic interest payments*

- *A Credit Default Swap specifies a stream of payments to firm* **A** *from* **C***, who usually owns B. If* ***O*** *defaults,* **C** *gives B to* **A***, and receives the principal & interest value of B*
- *An Asset Swap trades B from* **C** *to* **A***, and* **C** *pays LIBOR plus a spread.* **A** *pays par value, plus fixed coupons at the bond rate.*

Multifactor Model Form

- *Based on M stochastic drivers plus idiosyncratic risk, a company changes riskiness or even defaults*
- *Commonly, we consider a ratings migration model, with discrete riskiness*
- *CreditMetrics is the well-known example*
- *A default-only model would be a two-state migration model*

Sources Of Correlation Info

- *In the real world measure, we have time series of ratings, financial ratios, KMV EDFs (Expected Default Frequencies).*
- *Willing to believe asset correlations?*
- *In the pricing measure, we have fewer sources, plus recovery rate issues*
  - *Bond prices are notoriously dirty*
  - *A few years' CDS time series data.*

Measure Mapping

We could use data from the "wrong" measure if

- *We assumed correlation is unaffected by change of measure (essentially, that it is not priced by the market), or*
- *We had a means of mapping between actual and pricing measures by, say, estimating liquidity spread and market prices of the various risks*

Example: KMV Factor Model

- *Begin with a set of independent uniform normally distributed global drivers G_i*
- *These drive normal region and industry factors C_j, which also have idiosyncratic components c_j*
- *The C_j then drive normal variables X_n for each firm, with idiosyncratic components x_j*

- *Each firm's Xn depends on all the Cj and of course the Gi, in addition to its own idiosyncratic variable.*

$$X_n = x_n + \sum x_{nj} C_j = x_n + \sum x_{nj}\left(c_j + \sum c_{ji} G_i\right)$$

- *Multifactor models with independent firm-specific components allow for efficient simulation*
  - *Relatively small number of drivers even for large portfolios*
  - *Don't need singular value decomposition*

- *KMV normalizes the X_n to have unit variance.*
- *Each firm has a dependency coefficient R, such that x_n has variance $1-R^2$.*
- *If we need random variables from some other distribution A, e.g. Student's T, we can (usually) map it using the distribution functions*

$$Y_n = A^{-1}\left(N(X_n)\right)$$

Whence Factors?

- *To create a factor model, we must first choose factors, and have a way of obtaining dependence of our random variables on them*
- *Start with historical time series data*
- *Possible techniques include: hand-selecting factors (e.g. KMV regions and industries), doing principal component analysis, cluster analysis, stepwise regression, and so on*

- *KMV's approach is hybrid. The global factors are independent, and probably came from PCA. The intermediate factors are chosen by humans, with loadings a subjective matter*

- *A simpler factor model could choose, say, the first three principal components, and then for each firm set the loadings to the dot products*

- *But, still need to deal with firms not in the time series data set. Maybe use proxies.*

After the Factor

- *Once we have a factor model for stochastic drivers, we still need to choose a risk model*
  - *Ratings migration*
  - *Copulas*
  - *Spreads and defaults*

Ratings Migration

- *Each firm has a sequence of transition boundaries for X_n, endpoints of "buckets"*
- *Ratings 1... N_r from worst to best*
- *The new rating (let default=rating 0) is determined by which bucket X_n fell in*
- *Using a non-normal Y_n gives slightly different transition correlations*

Copula Models

- *A copula assumes we start with known default time distributions for the firms, and imposes a correlation structure on the default times*
- *We will see more of these in the afternoon*
- *Our factor model may be used to set the coefficients of the correlation structure in our copula function of interest.*

Spread Models

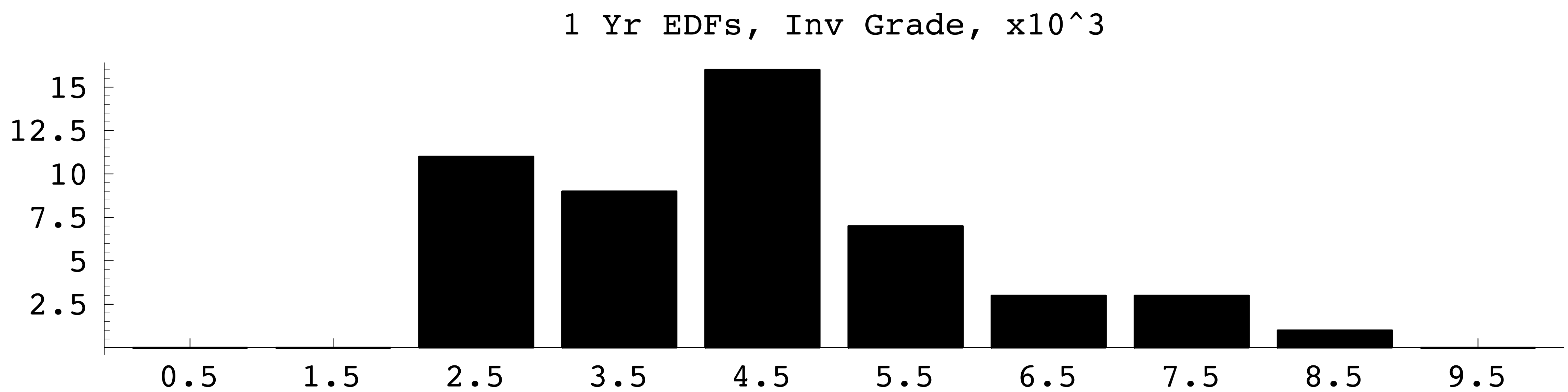
- *We can use our factor-derived variables to drive equations for the evolution of each firms spread (say, according to Ornstein-Uhlenbeck processes)*
- *Several possibilities exist for incorporating defaults*
- *Shouldn't spreads jump up if somebody defaults?*

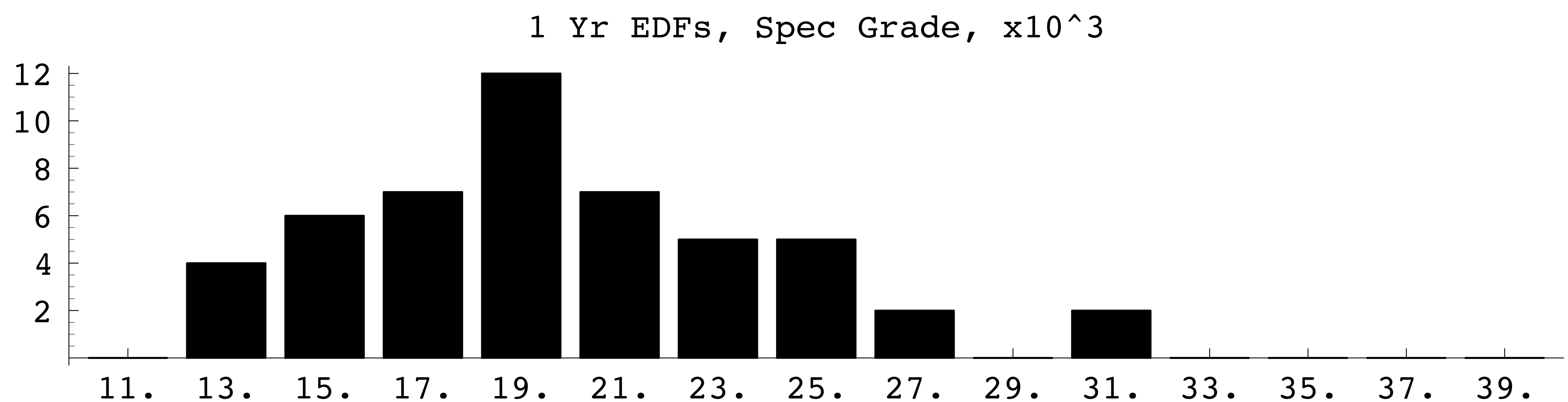
Fitting A Factor Model

- *Generally speaking, it very hard to fit correlation variables to credit or default data. Recovery rates and small sample counts interfere*
- *Easier to fit correlations to "equity" time series*
- *Transition matrix fits are not too hard*

EDF Distribution, Factor Migration

1000 Companies, Quarterly, 10 yrs w/ replacement





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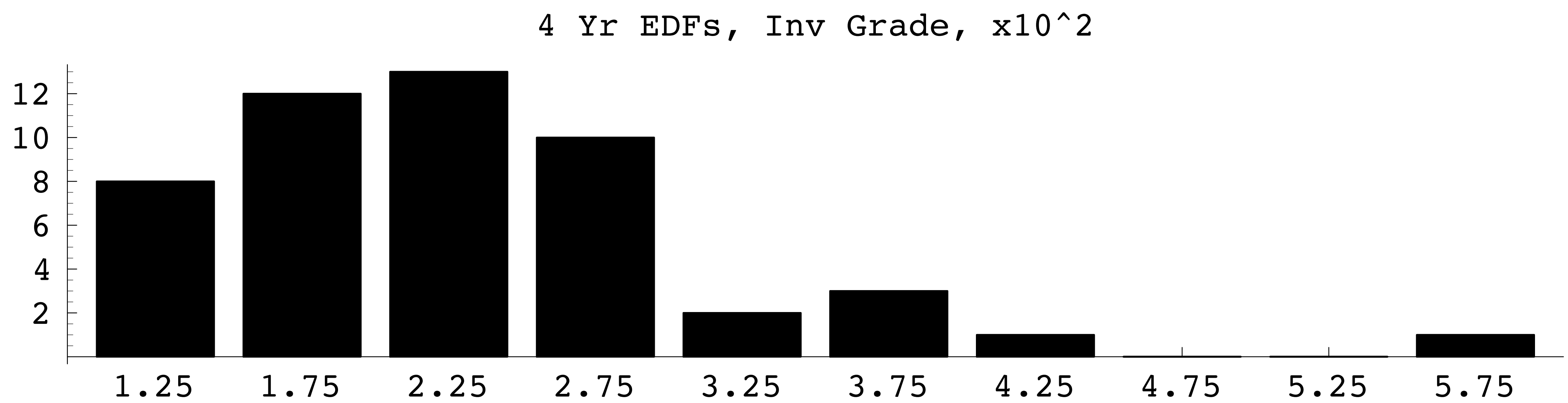
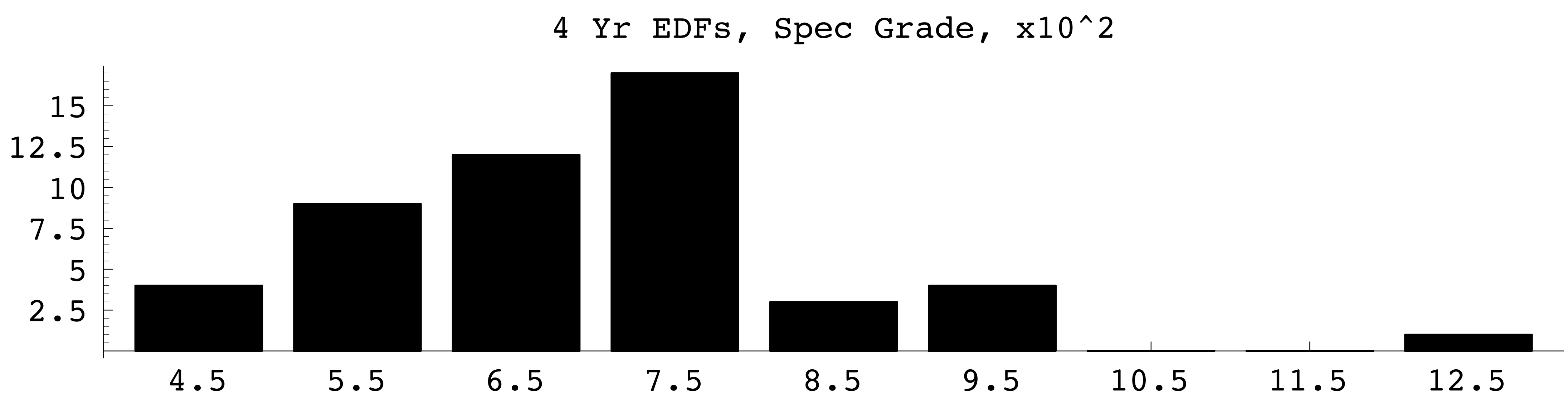




Credit Options

What Are Credit Options?

- *Basic Types:*
  - *Embedded bond or CDS options, tenor typically several years. Often knockout.*
  - *Options on CDS or asset swaps, tenor typically << 1yr. Asset swaptions are much less vanilla than the swaps.*
  - *Embedded convertible bond options (but spread dynamics are usually ignored)*

Credit Options Comments

- *Knockout clauses on CDS options make the accounting simple in case of default.*
- *Liquidity is highest in 1-5 month tenors*
- *Straddles are popular (as volatility plays or correlation hedges)*
- *Seniority (recovery rate) issues can interfere with the comparison of credit spreads in different instruments.*

Trading Motivation

- *Speculation on credit spreads or spread volatility*
- *Regulatory satisfaction*
- *Yield enhancement*
- *Hedging exposures (e.g. project finance loans)*

CDSwaption: Available Models

- *A Black (1976) formula serves vanillas:*
  - *Volatility skew corrects for distributional errors*
  - *Other models are significant only in that the terminal distribution differs. This is equivalent to a skew.*
- *Cancellable swaps require a tree, especially for interest rate correlation*

Black (1976) Applied

- *Consider pricing a knockout in the survival measure (Schönbucher) with trivial spread dynamics (flat, parallel shifts)*
- *Fwd spread* $\mathbf{s}$, *strike* $\mathbf{s_K}$ *spread vol* $\boldsymbol{\sigma}$, *call/put indicator* $\mathbf{g}$ *in {1,-1}, zero recovery zero-coupon bond prices* $\boldsymbol{B_d(t)}$, *underlying swap payments at times* $\boldsymbol{t_k}$. *Option value is*

$$\sum_k B_d(t_k) g\left(sN(gd_+) - s_K N(gd_-)\right), \quad d_\pm = \frac{\log \frac{s}{s_K} \pm \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}}$$

Hedging

- *Primary hedge DV01 (sensitivity to underlying spread). Use proxies at times.*
- *Also important for longer T: jump risk*
- *Trading both 1 and 5 year underlying tenors can do both, especially if spread curve has parallel shifts*
- *Push the hedges to the flow traders*

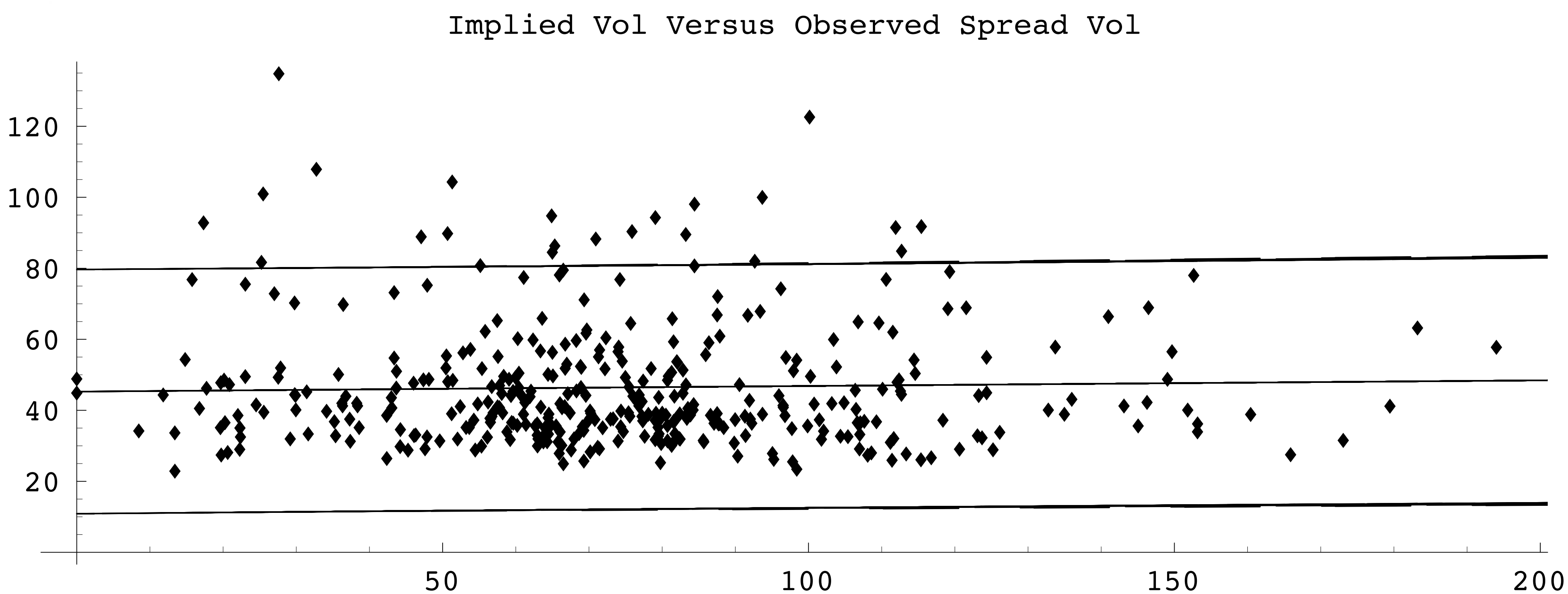
Structuring With Options

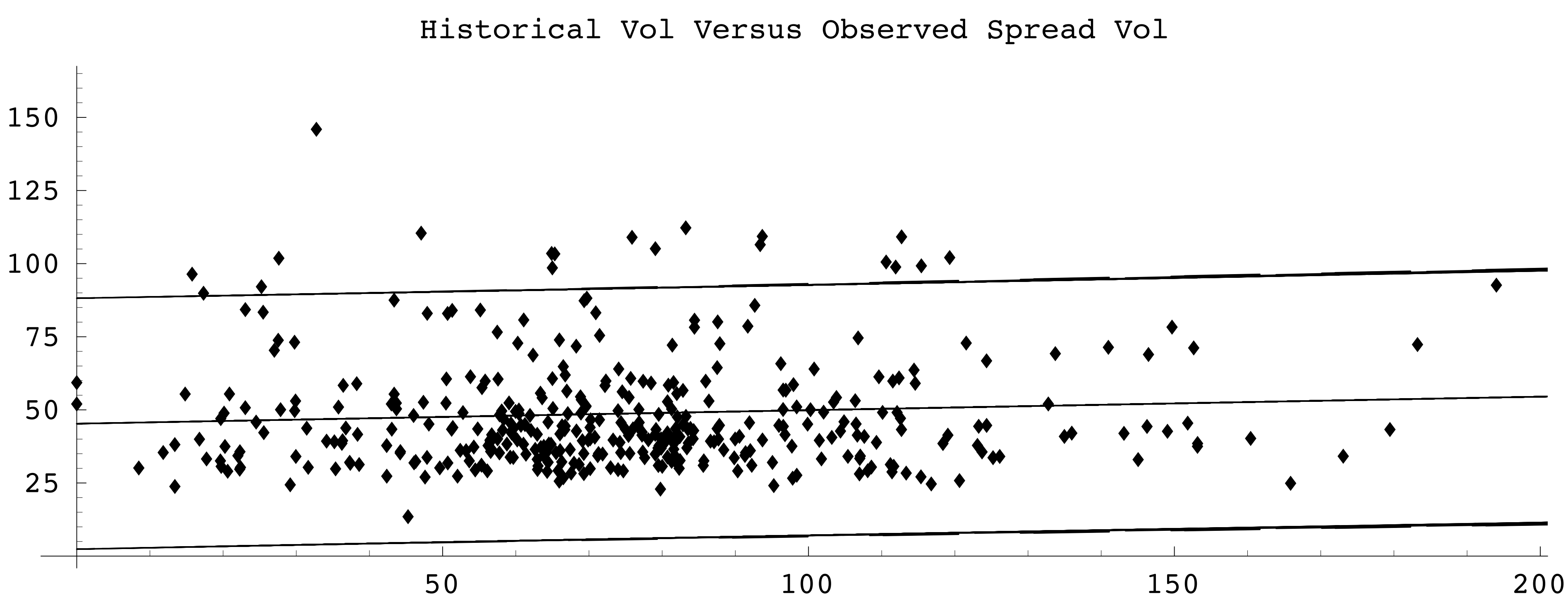
- *Demand for credit options comes from cancelable loans, project finance extensible loans, and other credit exposures of uncertain size.*
- *Correlation of credit spreads and interest rates can be important for these longer tenors.*

Obtaining a Volatility

- *More liquid names will have a satisfactory time series of spread data.*
- *Though skew should appear, it is basically unavailable from market data. Use another distribution (e.g. gamma)?*
- *Similar traded names as proxies*
- *Last ditch: guesstimate from equity vol*

But Equity Vol Is Not Enough!





Basket & Tranche Options

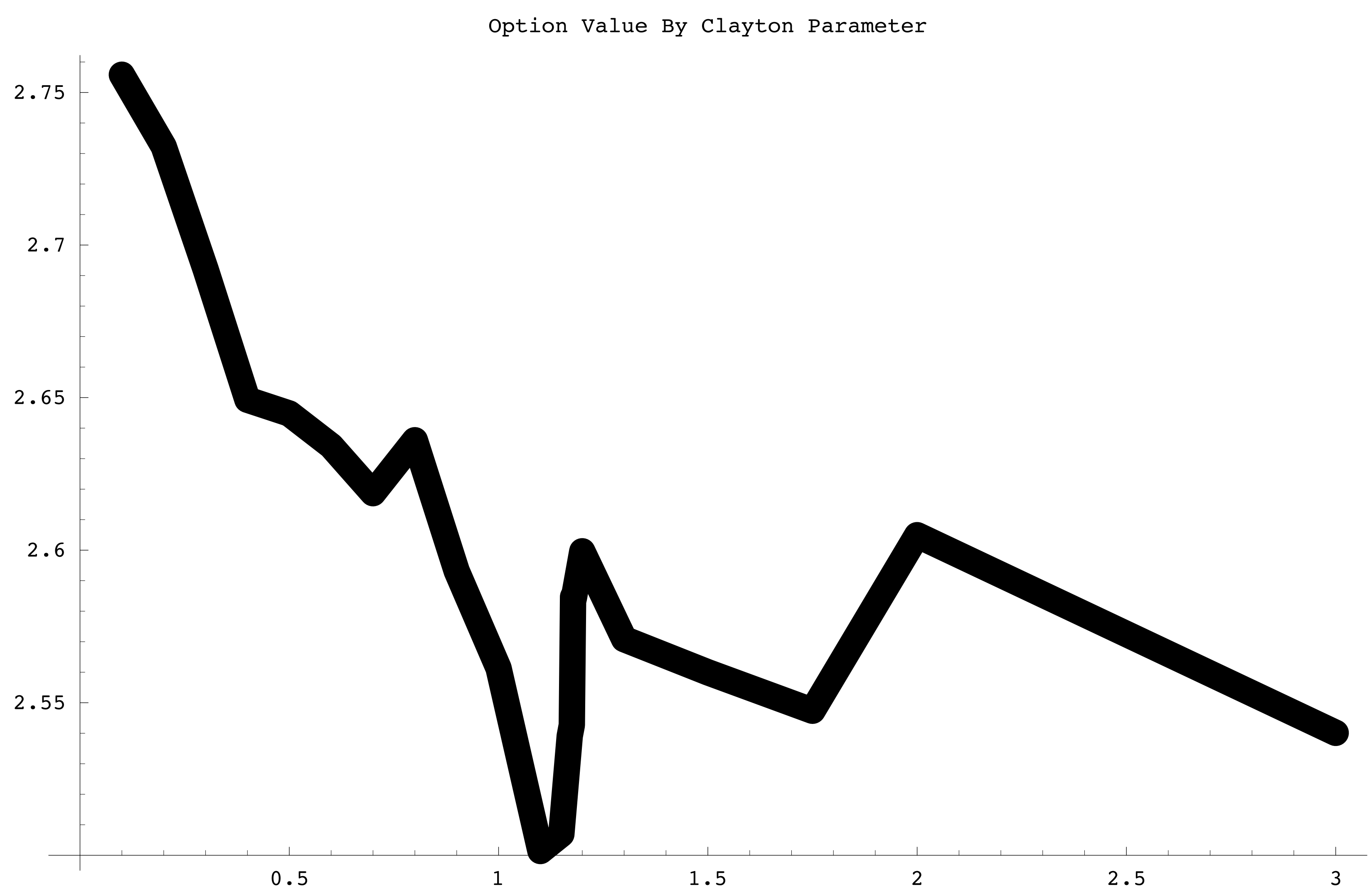
- *Just barely starting up; an active area of research*
- *Attractive to, say, reinsurers with variable tranche exposures*
- *Index credit swaptions*

Tranche Option Pricing

- *Want a multi-name spread and default model consistent with*
  - *single name default probability curves*
  - *default swaption prices*
  - *tranche prices*
- *Influential ideas: "usual" copulas have difficult conditionals. Schönbucher and Rogge use a generalized Archimedean copula (e.g. Clayton)*

The Effect Of Correlation

ATM TraXX basket option value by Clayton gamma factor



- *Good fitting versus overparameterization*
- *Correlation regimes*
- *Require a reasonable way for observed defaults to influence spreads of survivors*
- *Possible spread dynamics: Ornstein-Uhlenbeck (but what does a negative spread mean?!), Cox-Ingersoll-Ross*
- *Vol sources; knockout vol much lower*

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